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Flow and Heat Transfer of Micropolar and Viscous Fluid with Source or Sink in A Vertical Channel

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ABSTRACT: The flow nature for mixture of viscous and micro polar fluid in the presence of source or sink in a vertical channel is studied. The effect of governing parameter such as the ratio of Grashof number to Reynolds number, viscosity ratio, width ratio, pressure and material parameter show the similar nature keeping either source or sink the constant value. The effect of source is to increase, micro rotation velocity and temperature whereas sink is to decrease velocity, increase the micro rotation velocity and to decrease the temperature.

I. INTRODUCTION

Non-Newtonian fluids are of increasing importance in modern technology due to its growing use in many activities, such as molten plastic, paints, drilling and petroleum and polymer solution.

Eringen (1966) proposed the theory of micropolar fluids, which show micro rotation effects as well as micro-inertia. The theory of thermo micropolar fluids was developed by Eringen (1972) by extending his theory of micropolar fluids. The theory of micropolar fluid is generating a lot of interest and many classical flows are being re-examined to determine the effects of fluid microstructure. Gorla (1983) investigated the forced convective heat transfer to a micropolar fluid flowing over a flat plate. Free convective heat transfer to a micropolar fluid along a vertical plate was studied by Jena and Mathur (1981)

Much work has not been done on the fluid flow and heat transfer in the presence of heat generation or absorption effects. Examples of heat generation are flowing water in solar collector and radiative cooling of molten glass in a Fore hearth.

Therefore the objective of this chapter is to consider the problem of flow and heat transfer in a vertical enclosure consisting of the viscous and micropolar fluid in the presence of source or sink.

II. MATHEMATICAL FORMULATION

Consider a steady, laminar, incompressible micro polar fluid through vertical parallel plates in the presence of heat generation or absorption. The region $0 \le y \le h^{(1)}$ is occupied by a micro polar fluid of density $\rho^{(1)}$, viscosity $\mu^{(1)}$, and material parameter Ω. The region $-h^{(2)} \le y \le 0$ is occupied by viscous fluid of density $\rho^{(2)}$, viscosity $\mu^{(2)}$. The fluids are assumed to have constant properties except the density in the buoyancy term in the momentum equation. The fluid rises in the channel driven by buoyancy forces. The transport properties of both fluids are assumed to be constant. We assume the flow is steady, laminar and fully developed. It is also assumed that the viscous dissipation is neglected. Under these assumptions, the basic equations of the micro polar fluid are

Region-I

$$\left(\mu^{(1)}+\Omega\right)\frac{d^2u^{(1)}}{dy^2}+\Omega\frac{dN}{dy}+\rho^{(1)}g\beta^{(1)}\left(T_1-T_0\right)-\frac{\partial P}{\partial x} \tag{2.1}$$

$$\gamma\frac{d^2N}{dy^2}-2\Omega N-\Omega\frac{du^{(1)}}{dy}=0 \tag{2.2}$$

$$k^{(1)}\frac{d^2T^{(1)}}{dy^2}\pm Q^{(1)}\left(T_1-T_0\right)=0 \tag{2.3}$$

Region-II

$$\mu^{(2)}\frac{d^2u^{(2)}}{dy^2}+\rho^{(2)}g\beta^{(2)}\left(T_2-T_0\right)-\frac{\partial P}{\partial x}=0 \tag{2.4}$$

$$k^{(2)}\frac{d^2T^{(2)}}{dy^2} \pm Q^{(2)}\left(T_2 - T_0\right) = 0 \qquad (2.5)$$

The appropriate boundary and interface conditions on velocity in the mathematical form are:

$U^{(1)} = 0$ at $Y = h^{(1)}$

$U^{(2)} = 0$ at $Y = -h^{(2)}$

$U^{(1)} = U^{(2)}$ (Continuity of velocity) at $Y = 0$

$\left(\mu^{(1)} + \Omega\right)\frac{dU^{(1)}}{dy} + \Omega N = \mu^{(2)}\frac{dU^{(2)}}{dy}$ (Continuity of shear stress) at $Y = 0$ (2.6a)

The walls are maintained at constant different temperatures T_w and T_1 at $y = h^{(1)}$ and $y = -h^{(2)}$ respectively.

The boundary and interface conditions on temperature are

$T^{(1)} = T_W$ at $Y = h^{(1)}$

$T^{(2)} = T_1$ at $Y = -h^{(2)}$

$T^{(1)} = T^{(2)}$ (Continuity of temperature) at $Y = 0$

$\frac{dT^{(1)}}{dy}(0) = \frac{dT^{(2)}}{dy}(0)$ (Continuity of heat flux) at $Y = 0$ (2.6b)

Further we assume that γ has the following form as proposed by Ahmadi (1976) $\gamma = \left(\mu + \frac{\Omega}{2}\right)j$, where j is the micro inertia density.

It is convenient to non-dimensionalize the governing equations using the variables,

$$y^{(i)} = \frac{y^{(i)}}{h^{(i)}},\ U^{(i)} = \frac{u^{(i)}}{U},\ \theta^{(i)} = \frac{T_i - T_0}{Tw - T_s},$$

$$\bar{P} = \frac{\partial P/\partial x}{\mu^{(i)}u^{(i)}/h^{(i)2}},\ i = 1, 2$$

$$\bar{N} = \frac{U}{h}N\ ,\ \ \upsilon = \left(\mu^{(1)} + \frac{\Omega}{2}\right)j,$$

$$\kappa = \frac{\mu^{(1)}}{\Omega}\ ,\ j = h^{(1)\ 2}$$

$$m = \frac{\mu^{(1)}}{\mu^{(2)}},\quad h = \frac{h^{(2)}}{h^{(1)}},\ \ \rho = \frac{\rho^{(2)}}{\rho^{(1)}},\ \ \beta = \frac{\beta^{(2)}}{\beta^{(1)}} \qquad (2.7)$$

Where U is the characteristic velocity

Region-I

$$\left(1+\kappa\right)\frac{d^2u^{(1)}}{dy^2} + \kappa\frac{dN}{dy} + GR\,\theta^{(1)} + P = 0 \qquad (2.8)$$

$$\left(1+\frac{\kappa}{2}\right)\frac{d^2N}{dy^2} - 2\kappa N - \kappa\frac{du^{(1)}}{dy} = 0 \qquad (2.9)$$

$$\frac{d^2\theta^{(1)}}{dy^2} + \phi^{(1)}\theta^{(1)} = 0 \qquad (2.10)$$

Region-II

$$\frac{d^2u^{(2)}}{dy^2} + \beta mrh^2\,GR\,\theta^{(2)} + mh^2P = 0 \qquad (2.11)$$

$$\frac{d^2\theta^{(2)}}{dy^2} + \phi^{(2)}\theta^{(2)} = 0 \qquad (2.12)$$

The non-dimensional form of the velocity, temperature boundary and interface conditions becomes

$N = 0$ at $y = 1$

$\frac{dN}{dy} = 0$ at $y = 0$ (2.13a)

$u^{(1)} = 0$; at $y = 1$

$u^{(2)} = 0$; at $y = -1$

$u^{(1)} = u^{(2)}$ at $y = 0$

$\left(1+\kappa\right)\frac{du^{(1)}}{dy} + \kappa N = \frac{1}{mh}\frac{du^{(2)}}{dy}$ at $y = 0$ (2.13b)

$\theta^{(1)} = 1$ at $y = 1$

$\theta^{(2)} = 0$ at $y = -1$

$\theta^{(1)}\left(0\right) = \theta^{(2)}\left(0\right)$ at $y = 0$

$\frac{d\theta^{(1)}}{dy}(0) = \frac{1}{kh}\frac{d\theta^{(2)}}{dy}(0)$ at $y = 0$ (2.13c)

III. SOLUTIONS

Solving analytically equations (2.8) to (2.12) using boundary and interface conditions as given by equations 2.13 (a, b, c) are

Heat-Generation Case $(+\phi^{(1)} > 0, +\phi^{(2)} > 0)$

Region-I

$$\theta^{(1)} = C_1 \cos\sqrt{\phi^{(1)}}\, y + C_2 \sin\sqrt{\phi^{(1)}}\, y$$

$$N = C_5 \cosh ay + C_6 \sinh ay + d_1 \sin\sqrt{\phi^{(1)}}\, y$$
$$+ d_2 \cos\sqrt{\phi^{(1)}}\, y + d_3 y + d_4 A$$

$$u^{(1)} = d_5 C_5 \sinh ay + d_5 C_6 \cosh ay + l_1 \cos\sqrt{\phi^{(1)}}\, y$$
$$+ l_2 \sin\sqrt{\phi^{(1)}}\, y + l_3 y^2 - C_7 y - C_8$$

Region-II

$$\theta^{(2)} = C_3 \cos\sqrt{\phi^{(2)}}\, y + C_4 \sin\sqrt{\phi^{(2)}}\, y$$

$$u^{(2)} = l_4 \cos\sqrt{\phi^{(2)}}\, y + l_5 \sin\sqrt{\phi^{(2)}}\, y + l_6 y^2 - C_9 y - C_{10}$$

Heat-Absorption Case $\left(+\phi^{(1)} < 0, +\phi^{(2)} < 0\right)$

Region-I

$$\theta^{(1)} = C_1 \cosh\sqrt{\phi^{(1)}}\, y + C_2 \sinh\sqrt{\phi^{(1)}}\, y$$

$$N = C_5 \cosh ay + C_6 \sinh ay + d_1 \sinh\sqrt{\phi^{(1)}}\, y$$
$$+ d_2 \cosh\sqrt{\phi^{(1)}}\, y + d_3 y + d_4 A$$

$$u^{(1)} = d_5 C_5 \sinh ay + d_5 C_6 \cosh ay$$
$$+ l_1 \cos\sqrt{\phi^{(1)}}\, y + l_2 \sin\sqrt{\phi^{(1)}}\, y + l_3 y^2 - C_7 y - C_8$$

Region-II

$$\theta^{(2)} = C_3 \cosh\sqrt{\phi^{(2)}}\, y + C_4 \sinh\sqrt{\phi^{(2)}}\, y$$

$$u^{(2)} = l_4 \cosh\sqrt{\phi^{(2)}}\, y + l_5 \sinh\sqrt{\phi^{(2)}}\, y + l_6 y^2$$
$$- C_9 y - C_{10}$$

IV. RESULTS AND DISCUSSION

An exact solution for flow and heat transfer of micro polar and viscous fluid in a vertical channel in the presence of source or sink is analyzed. Solutions are given by equations (2.13) to (2.17) for heat generation and (2.18) to (2.22) for heat absorption.

The results are presented graphically in figures 1 to 12 for velocity for a source and 13 to 24 on velocity for a sink. Figures 25 and 26 display the effect of width ratio and source or sink on temperature.

We observe that effect of GR, viscosity ratio m, width ratio h, pressure gradient P and material parameter remains the same as explained in the previous chapter for heat generation (Figs. 1 to 10). Effect of heat generation Φ on velocity and micro rotation velocity is to increase the velocity and the magnitude is large for viscous fluid compared to micro polar fluid as seen in figures 11 and 12.

Figure 13 shows the effect of GR on velocity u for heat absorption which remains same as that of heat generation. Figure 14 show the effect of GR on micro rotation velocity N for heat absorption, we observe that these profiles for source (Fig. 2) are exactly the image of sink (Fig. 14). The effect of viscosity ratio m on velocity u and micro rotation velocity N is to increase the velocity and the magnitude is large for viscous fluids compared to micro polar fluid as seen in figures 15 and 16

Effect of width ratio h, pressure P on velocity u and micro rotation velocity N for heat absorption show the similar nature as that for heat generation (Figs. 17 to 20) except the difference in magnitude. The effect of material parameter on velocity u and micro rotation velocity N for a sink also show the similar nature as that of source (Figs. 21 and 22). The effect of heat absorption is to decrease the velocity as Φ increases (Figs. 23 and 24). Figures 25 and 26 shows that temperature increases as width ratio increases. Figures 27 and 28 shows that as heat generation Φ increases temperature increases where as temperature decreases as heat absorption increases.

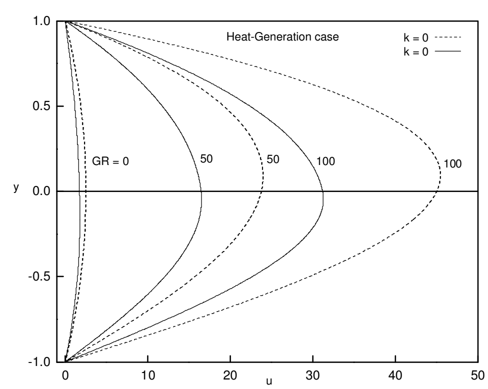

Fig. 1 Effect of GR on velocity profiles u

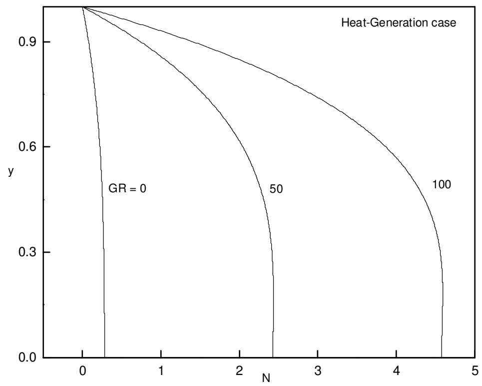

Fig. 2 Effect of GR on micro rotation velocity profiles N

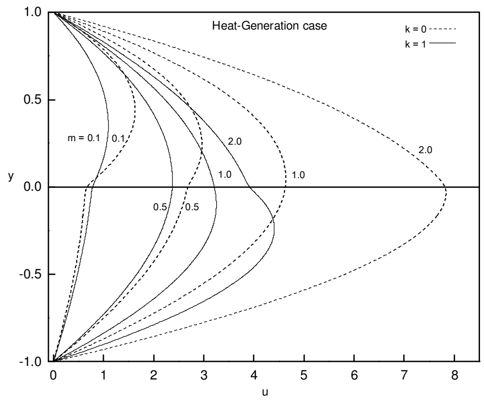

Fig. 3 Effect of viscosity ratio m on the velocity profiles u

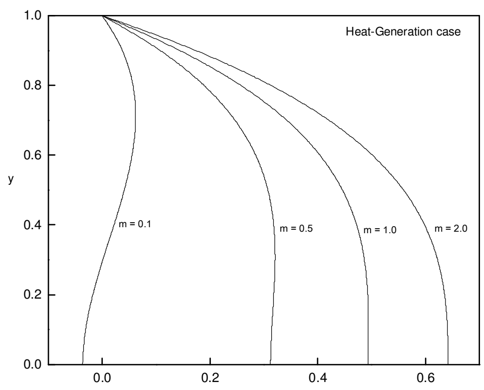

Fig. 4 Effect of viscosity ratio m on microrotation velocity profiles N

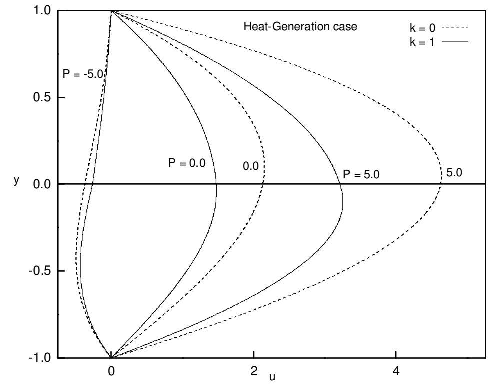

Fig. 7 Effect of Pressure gradient P on velocity profiles u

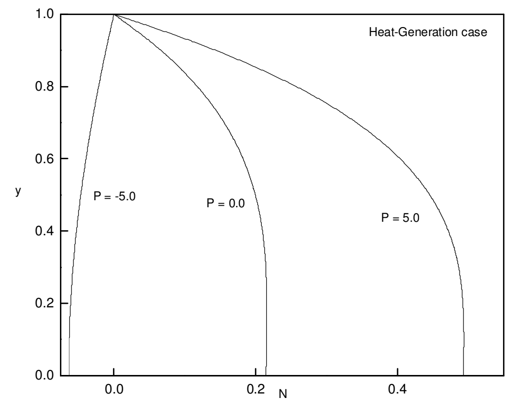

Fig. 8 Effect of Pressure gradient P on microrotation velocity profiles N

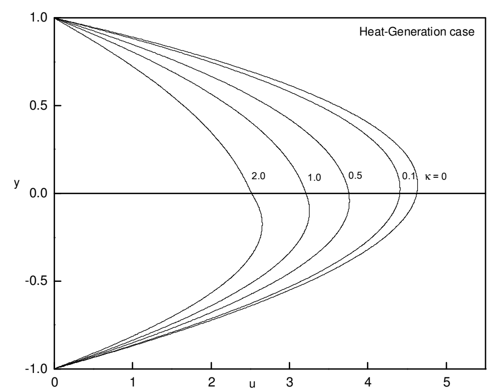

Fig. 9 Effect of material parameter κ on the velocity profiles u

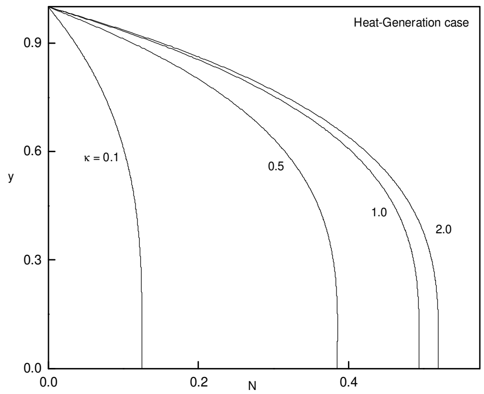

Fig. 10 Effect of material parameter κ on microrotation velocity profiles N

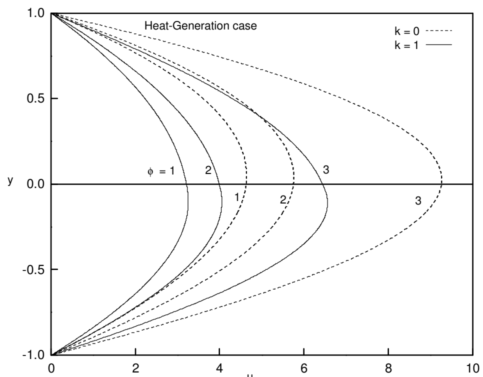

Fig. 11 Effects of ϕ on velocity profiles u

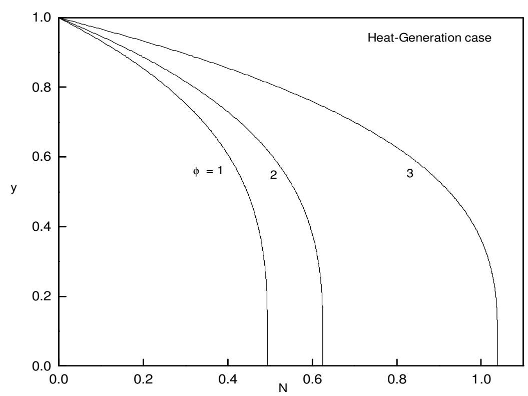

Fig. 12 Effect of ϕ on microrotation velocity profiles N

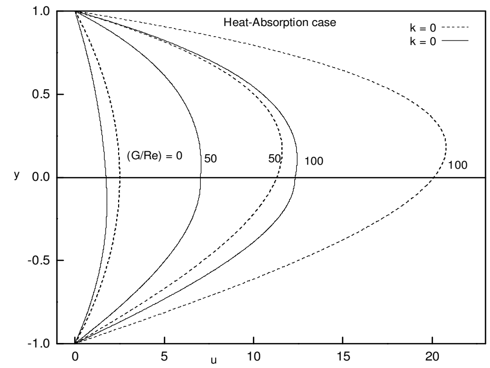

Fig. 13 Effect of GR on velocity profiles u

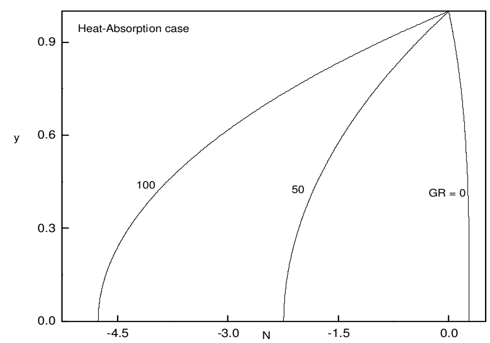

Fig. 14 Effect of GR on microrotation velocity profiles N

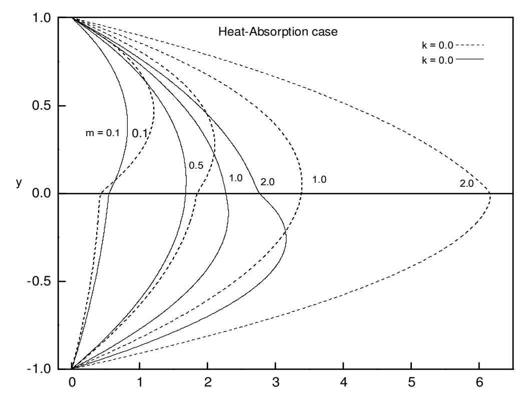

Fig. 15 Effect of viscosity ratio m on velocity profiles u

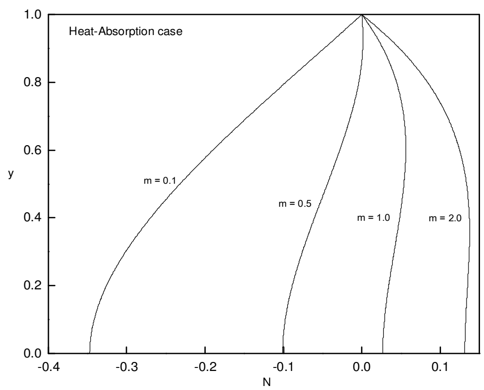

Fig. 16 Effect of viscosity ratio m on microrotation velocity profiles N

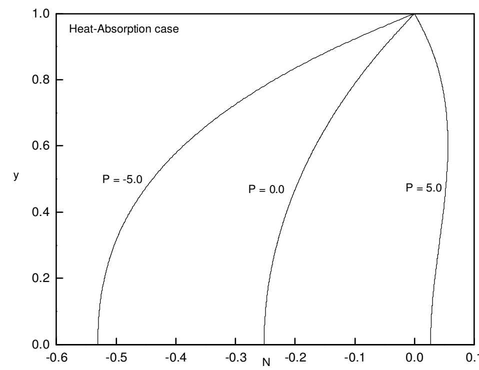

Fig. 20 Effect of Pressure gradient P on microrotation velocity profiles N

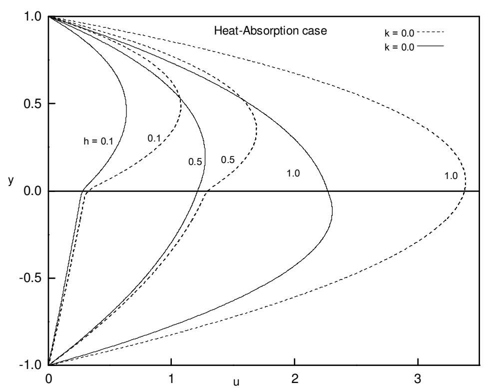

Fig. 17 Effect of width ratio h on the velocity profiles u

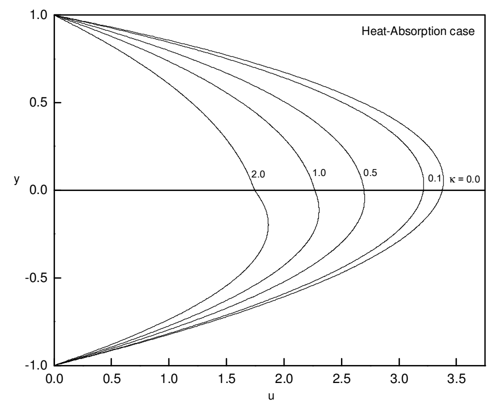

Fig. 21 Effect of material parameter κ on velocity profiles u

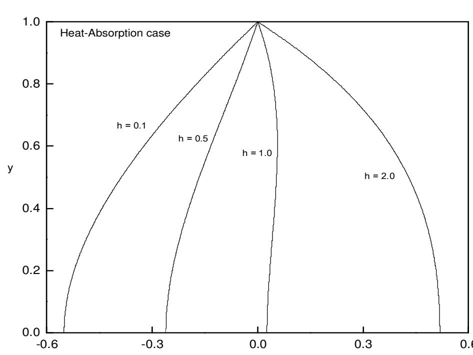

Fig. 18 Effect of width ratio h on microrotation velocity profiles N

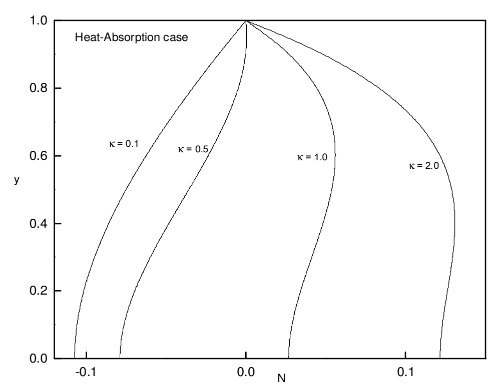

Fig. 22 Effect of material parameter κ on velocity profiles N

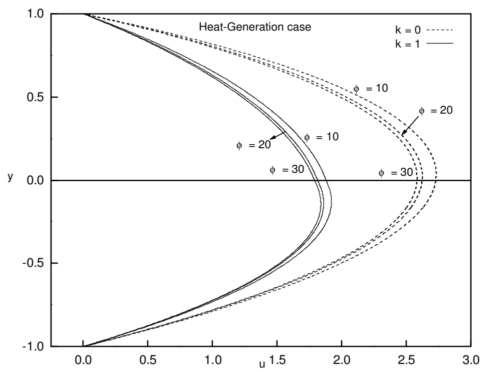

Fig. 23 Effect of ϕ on velocity profiles u

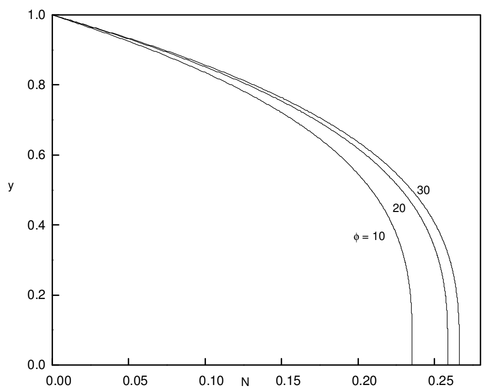

Fig. 24 Effect of ϕ on microrotation velocity profiles N

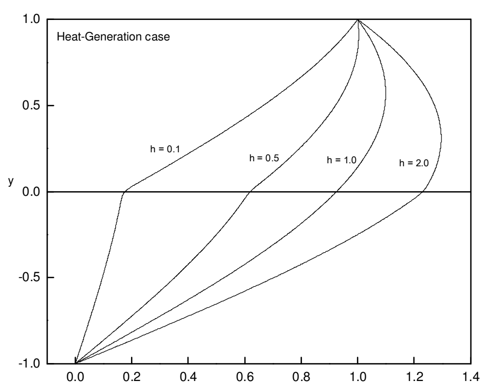

Fig. 25 Effect of width ratio h on temperature profiles

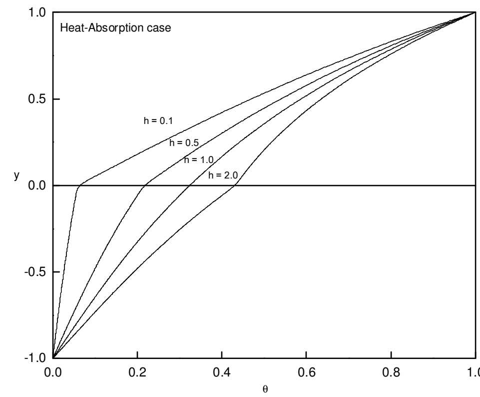

Fig. 26 Effect of width ratio h on temperaure profiles

Where
(for heat generation case):

$$C_4 = \left(k\,h\sqrt{\phi^{(1)}}\cos\sqrt{\phi^{(2)}}\right) \Big/ \left(\begin{array}{l}\sqrt{\phi^{(2)}}\sin\sqrt{\phi^{(1)}}\cos\sqrt{\phi^{(2)}} + \\ k\,h\sqrt{\phi^{(1)}}\sin\sqrt{\phi^{(2)}}\cos\sqrt{\phi^{(1)}}\end{array}\right)$$

$$C_3 = C_4 \sin\sqrt{\phi^{(2)}} \Big/ \cos\sqrt{\phi^{(2)}}\ ; \quad C_1 = C_3;$$

$$C_2 = \frac{1}{\sin\sqrt{\phi^{(1)}}} - C_1 \frac{\cos\sqrt{\phi^{(1)}}}{\sin\sqrt{\phi^{(1)}}}\ \ a^2 = \frac{2\kappa}{(1+\kappa)};$$

$$\mathrm{GR} = \frac{\mathrm{G}}{\mathrm{Re}};$$

$$d_1 = 2\kappa C_1 GR \Big/ \left(\sqrt{\phi^{(1)}}\left(\phi^{(1)} + a^2\right)(2+\kappa)(1+\kappa)\right)$$

$$d_2 = -2\kappa C_2 GR \Big/ \left(\sqrt{\phi^{(1)}}\left(\phi^{(1)} + a^2\right)(2+\kappa)(1+\kappa)\right)$$

$$d_3 = P/(2+\kappa)\ ,\quad d_4 = (1+\kappa)/(2+\kappa);$$

$$C_6 = -\frac{1}{a}\left(d_1\sqrt{\phi^{(1)}} + d_3\right)$$

$$C_5c = -\left(C_6 \sinh a + d_1 \sin\sqrt{\phi^{(1)}} + d_2 \cos\sqrt{\phi^{(1)}} + d_3\right)/\cosh a$$

$$C_5cc = -d_4/\cosh a\ ,\quad C_5 = C_5c + C_5cc\,A$$

$$d_5 = -\frac{\kappa}{a(1+\kappa)};$$

$$l_1 = \frac{d_1\kappa}{(1+\kappa)\sqrt{\phi^{(1)}}} + \frac{C_1 GR}{\phi^{(1)}(1+\kappa)}$$

$$l_2 = -\frac{d_2\kappa}{(1+\kappa)\sqrt{\phi^{(1)}}} + \frac{C_2 GR}{\phi^{(1)}(1+\kappa)};$$

$$l_3 = -\frac{d_3\kappa}{2(1+\kappa)} - \frac{P}{2(1+\kappa)}$$

$$l_4 = \frac{\beta mrh^2\, GR\, C_5}{\phi^{(2)}};$$

$$l_5 = \frac{\beta mrh^2\, GR\, C_4}{\phi^{(2)}};$$

$$l_6 = -\frac{mh^2 P}{2}$$

$$C_7 c = \frac{mh}{1+mh(1+\kappa)}\left[\frac{1}{mh}\begin{pmatrix} d_5 C_5 c \sinh a + d_5 C_6 \cosh a + \\ l_1 \cos\sqrt{\phi^{(1)}} + l_2 \sin\sqrt{\phi^{(1)}} + l_3 - \\ d_5 C_6 - l_1 + l_4 \end{pmatrix}\right.$$

$$-\frac{1}{mh}\left(l_4 \cos\sqrt{\phi^{(2)}} - l_5 \sin\sqrt{\phi^{(2)}} + l_6\right) +$$

$$\left.(1+\kappa) l_2 \sqrt{\phi^{(1)}} + \kappa d_2 - \frac{l_5\sqrt{\phi^{(2)}}}{mh}\right]$$

$$C_7 cc = \frac{mh}{1+mh(1+\kappa)}\left[\frac{1}{mh}\left(d_5 C_5 cc \sinh a\right) + \kappa d_4\right]$$

$$C_7 = C_7 c + C_7 cc\, A$$

$$A = -\frac{C_7 c(2+\kappa)}{\kappa + (2+\kappa) - 2C_9(2+\kappa)}$$

Where (for heat absorption case):

$$C_3 = \left(k\, h\sqrt{\phi^{(1)}} \sinh\sqrt{\phi^{(2)}}\right) \Big/ \begin{pmatrix} kh\sqrt{\phi^{(1)}} \sin\sqrt{\phi^{(1)}} \cosh\sqrt{\phi^{(1)}} + \\ \sqrt{\phi^{(2)}} \sinh\sqrt{\phi^{(1)}} \cosh\sqrt{\phi^{(2)}} \end{pmatrix}$$

$$C_1 = C_3;\; C_4 = C_3 \cos\sqrt{\phi^{(2)}} \Big/ \sin\sqrt{\phi^{(2)}};$$

$$C_2 = \frac{1}{\sinh\sqrt{\phi^{(1)}}} - \frac{C_1 \cosh\sqrt{\phi^{(1)}}}{\sinh\sqrt{\phi^{(1)}}}$$

$$a^2 = \frac{2\kappa}{(1+\kappa)}$$

$$d_1 = 2\kappa C_1 GR \Big/ \left(\sqrt{\phi^{(1)}}\left(\phi^{(1)} + a^2\right)(2+\kappa)(1+\kappa)\right)$$

$$d_2 = 2\kappa C_2 GR \Big/ \left(\sqrt{\phi^{(1)}}\left(\phi^{(1)} + a^2\right)(2+\kappa)(1+\kappa)\right)$$

$$d_3 = P/(2+\kappa)$$

$$d_4 = (1+\kappa)/(2+\kappa);$$

$$C_6 = -\frac{1}{a}\left(d_1\sqrt{\phi^{(1)}} + d_3\right)$$

$$C_5 c = -\begin{pmatrix} C_6 \sinh a + d_1 \sinh\sqrt{\phi^{(1)}} + \\ d_2 \cosh\sqrt{\phi^{(1)}} + d_3 \end{pmatrix} / \cosh a$$

$$C_5 cc = -d_4 / \cosh a$$

$$C_5 = C_5 c + C_5 cc\, A; \quad d_5 = -\frac{\kappa}{a(1+\kappa)};$$

$$l_1 = -\frac{d_1\kappa}{(1+\kappa)\sqrt{\phi^{(1)}}} - \frac{C_1 GR}{\phi_1(1+\kappa)}$$

$$l_2 = -\frac{d_2\kappa}{(1+\kappa)\sqrt{\phi^{(1)}}} - \frac{C_2\, GR}{\phi^{(1)}(1+\kappa)};$$

$$l_3 = -\frac{d_3\kappa}{2(1+\kappa)} - \frac{P}{2(1+\kappa)}$$

$$l_4 = -\frac{\beta mrh^2\, GR\, C_5}{\phi^{(2)}};\; l_5 = -\frac{\beta mrh^2\, GR\, C_4}{\phi^{(2)}};$$

$$l_6 = -\frac{mh^2 P}{2}$$

$$C_{10} c = \frac{mh}{1+mh(1+\kappa)}\left[(1+\kappa)\begin{pmatrix} d_5 C_5 c \sinh a \\ + d_5 C_6 \cosh a + \\ l_1 \cosh\sqrt{\phi^{(1)}} + l_2 \sinh\sqrt{\phi^{(1)}} \end{pmatrix} + l_3 - d_5 C_6\right.$$

$$-l_1 + l_4) + \frac{1}{mh}\begin{pmatrix} l_4 \cos\sqrt{\phi^{(2)}} - \\ l_5 \sin\sqrt{\phi^{(2)}} + l_6 \end{pmatrix} -$$

$$\left.(1+\kappa) l_2 \sqrt{\phi^{(2)}} - \kappa d_2 + \frac{l_5\sqrt{\phi^{(2)}}}{mh}\right]$$

$$C_{10} cc = \frac{mh}{1+mh(1+\kappa)}\left[(1+\kappa)\left(d_5 C_5 cc \sinh a\right) - \kappa d_4\right]$$

$$C_{10} = C_{10}c + C_{10}cc\,A \;\; C_8 = C_{10} - l_4 + l_1 + d_5 C_6 \,;$$

$$C_9 = C_{10} - l_4 \cosh\sqrt{\phi^{(2)}} + l_5 \sinh\sqrt{\phi^{(2)}} - l_6$$

$$C_7 c = d_5 C_5 c \sinh a + d_5 C_6 \cosh a +$$

$$l_1 \cosh\sqrt{\phi^{(1)}} + l_2 \sinh\sqrt{\phi^{(1)}} + l_3 + l_4$$

$$-l_1 - d_5 C_6 - C_{10}c$$

$$C_7 cc = d_5 C_5 cc \sinh a - C_{10}cc \,;$$

$$C_7 = C_7 c + C_7 cc\,A$$

$$A = -\frac{C_7 c\left(1+\kappa\right)}{\kappa d_4 + \left(1+\kappa\right) - C_7 cc\left(1+\kappa\right)}$$

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